

CALCULUS BC

SECTION I

Time—1 hour and 30 minutes

Number of questions—40

Percent of total grade—50

Part A consists of 28 questions that will be answered on side 1 of the answer sheet. Following are the directions for Section I, Part A.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

- (1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $7 = xy - e^{xy}$, then $\frac{dy}{dx} =$

(A) $x - e^y$

(B) $y - e^x$

(C) $\frac{ye^{xy} + y}{x - xe^{xy}}$

(D) $\frac{-y}{x}$

(E) $\frac{ye^{xy} + y}{x + xe^{xy}}$

2. The volume of the solid that results when the area between the curve $y = e^x$ and the line $y = 0$, from $x = 1$ to $x = 2$, is revolved around the x -axis is

(A) $2\pi(e^4 - e^2)$

(B) $\frac{\pi}{2}(e^4 - e^2)$

(C) $\frac{\pi}{2}(e^2 - e)$

(D) $2\pi(e^2 - e)$

(E) $2\pi e^2$

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3. $\int \frac{x - 18}{(x + 3)(x - 4)} dx =$

(A) $\int \frac{5dx}{(x + 3)(x - 4)}$

(B) $\int \frac{dx}{(x + 3)(x - 4)}$

(C) $\int \frac{3dx}{x + 3} + \int \frac{2dx}{x - 4}$

(D) $\int \frac{15dx}{x + 3} - \int \frac{14dx}{x - 4}$

(E) $\int \frac{3dx}{x + 3} - \int \frac{2dx}{x - 4}$

4. If $y = 5x^2 + 4x$ and $x = \ln t$ then $\frac{dy}{dt} =$

(A) $\frac{10}{t} + 4$

(B) $10t \ln t + 4t$

(C) $\frac{10 \ln t + 4}{t}$

(D) $\frac{5}{t^2} + \frac{4}{t}$

(E) $10 \ln t + \frac{4}{t}$

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5. $\int_0^{\frac{\pi}{2}} \sin^5 x \cos x dx =$

(A) $\frac{1}{6}$

(B) $-\frac{1}{6}$

(C) 0

(D) -6

(E) 6

6. The tangent line to the curve $y = x^3 - 4x + 8$ at the point $(2, 8)$ has an x -intercept at

(A) $(-1, 0)$

(B) $(1, 0)$

(C) $(0, -8)$

(D) $(0, 8)$

(E) $(8, 0)$

7. The graph in the xy -plane represented by $x = 3\sin(t)$ and $y = 2\cos(t)$ is

(A) a circle

(B) an ellipse

(C) a hyperbola

(D) a parabola

(E) a line

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8. $\int \frac{dx}{\sqrt{4-9x^2}} =$

(A) $\frac{1}{6} \sin^{-1}\left(\frac{3x}{2}\right) + C$

(B) $\frac{1}{2} \sin^{-1}\left(\frac{3x}{2}\right) + C$

(C) $6 \sin^{-1}\left(\frac{3x}{2}\right) + C$

(D) $3 \sin^{-1}\left(\frac{3x}{2}\right) + C$

(E) $\frac{1}{3} \sin^{-1}\left(\frac{3x}{2}\right) + C$

9. $\lim_{x \rightarrow \infty} 4x \sin\left(\frac{1}{x}\right)$ is

(A) 0

(B) 2

(C) 4

(D) 4π

(E) nonexistent

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10. The position of a particle moving along the x-axis at time t is given by $x(t) = e^{\cos(2t)}$, $0 \leq t \leq \pi$. For which of the following values of t will $x'(t) = 0$?

I. $t = 0$

II. $t = \frac{\pi}{2}$

III. $t = \pi$

- (A) I only (B) II only (C) I and III only (D) I and II only (E) I, II, and III

11. $\lim_{h \rightarrow 0} \frac{\sec(\pi + h) - \sec(\pi)}{h} =$

(A) -1

(B) 0

(C) $\frac{1}{\sqrt{2}}$

(D) 1

(E) $\sqrt{2}$

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12. Use differentials to approximate the change in the volume of a cube when the side is decreased from 8 to 7.99 cm. (in cm^3)

- (A) 19.2
(B) 15.36
(C) 1.92
(D) 0.01
(E) 0.0001

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13. The radius of convergence of $\sum_{n=1}^{\infty} \frac{a^n}{(x+2)^n}$; $a > 0$ is

- (A) $(a-2) \leq x \leq (a+2)$
- (B) $(a-2) < x < (a+2)$
- (C) $(-a-2) > x > (a-2)$
- (D) $(a-2) > x > (-a-2)$
- (E) $(a-2) \leq x \leq (-a-2)$

14. $\int_0^1 \sin^{-1}(x) dx =$

- (A) 0 (B) $\frac{\pi+2}{2}$ (C) $\frac{\pi-2}{2}$ (D) $\frac{\pi}{2}$ (E) $\frac{-\pi}{2}$

15. The equation of the line *normal* to $y = \sqrt{\frac{5-x^2}{5+x^2}}$ at $x = 2$ is

- (A) $81x - 60y = 142$
- (B) $81x + 60y = 182$
- (C) $20x + 27y = 49$
- (D) $20x + 27y = 31$
- (E) $81x - 60y = 182$

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16. If c satisfies the conclusion of the Mean Value Theorem for Derivatives for $f(x) = 2 \sin x$ on the interval $[0, \pi]$ then c could be
- (A) 0
(B) $\frac{\pi}{4}$
(C) $\frac{\pi}{2}$
(D) π
(E) There is no value of c on $[0, \pi]$
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17. The average value of $f(x) = x \ln x$ on the interval $[1, e]$ is

- (A) $\frac{e^2 + 1}{4}$ (B) $\frac{e^2 + 1}{4(e + 1)}$ (C) $\frac{e + 1}{4}$ (D) $\frac{e^2 + 1}{4(e - 1)}$ (E) $\frac{3e^2 + 1}{4(e - 1)}$
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18. A 17-foot ladder is sliding down a wall at a rate of -5 feet/sec. When the top of the ladder is 8 feet from the ground, how fast is the foot of the ladder sliding away from the wall (in feet/sec.)?

- (A) $\frac{75}{8}$ (B) $\frac{8}{3}$ (C) $\frac{3}{8}$ (D) -16 (E) $\frac{-75}{3}$
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19. If $\frac{dy}{dx} = 3y \cos x$, and $y = 8$ when $x = 0$, then $y =$

(A) $8e^{3\sin x}$

(B) $8e^{3\cos x}$

(C) $8e^{3\sin x} + 3$

(D) $3\frac{y^2}{2}\cos x + 8$

(E) $3\frac{y^2}{2}\sin x + 8$

20. The length of the curve determined by $x = 3t$ and $y = 2t^2$ from $t = 0$ to $t = 9$ is

(A) $\int_0^9 \sqrt{9t^2 + 4t^4} dt$

(B) $\int_0^{162} \sqrt{9 - 16t^2} dt$

(C) $\int_0^{162} \sqrt{9 + 16t^2} dt$

(D) $\int_0^3 \sqrt{9 - 16t^2} dt$

(E) $\int_0^9 \sqrt{9 + 16t^2} dt$

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21. If a particle moves in the xy -plane so that at time $t > 0$ its position vector is (e^{t^2}, e^{-t^3}) then its velocity at time $t = 3$ is

- (A) $(\ln 6, \ln(-27))$
- (B) $(\ln 9, \ln(-27))$
- (C) (e^9, e^{-27})
- (D) $(6e^9, -27e^{-27})$
- (E) $(9e^9, -27e^{-27})$

22. The graph of $f(x) = \sqrt{11+x^2}$ has a point of inflection at

- (A) $(0, \sqrt{11})$
- (B) $(-\sqrt{11}, 0)$
- (C) $(0, -\sqrt{11})$
- (D) $\left(\sqrt{\frac{11}{2}}, \sqrt{\frac{33}{2}}\right)$
- (E) There is no point of inflection.

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23. What is the volume of the solid generated by rotating about the y -axis the region enclosed by $y = \sin x$ and the x -axis, from $x = 0$ to $x = \pi$?

(A) π^2 (B) $2\pi^2$ (C) $4\pi^2$ (D) 2 (E) 4

24. $\int_{\frac{2}{\pi}}^{\infty} \frac{\sin\left(\frac{1}{t}\right)}{t^2} dt =$

(A) 1 (B) 0 (C) -1 (D) 2 (E) Undefined

25. A rectangle is to be inscribed between the parabola $y = 4 - x^2$ and the x -axis, with its base on the x -axis. A value of x that maximizes the area of the rectangle is

(A) 0 (B) $\frac{2}{\sqrt{3}}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$ (E) $\frac{\sqrt{3}}{2}$

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26. $\int \frac{dx}{\sqrt{9-x^2}} =$

- (A) $\sin^{-1} 3x + C$
(B) $\ln|x + \sqrt{9-x^2}| + C$
(C) $\frac{1}{3} \sin^{-1} x + C$
(D) $\sin^{-1} \frac{x}{3} + C$
(E) $\frac{1}{3} \ln|x + \sqrt{9-x^2}| + C$

27. Find $\lim_{x \rightarrow \infty} x^{\frac{1}{x}}$

- (A) 0 (B) 1 (C) ∞ (D) -1 (E) $-\infty$

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28. What is the sum of the Maclaurin series $\pi - \frac{\pi^3}{3!} + \frac{\pi^5}{5!} - \frac{\pi^7}{7!} + \dots + (-1)^n \frac{\pi^{2n+1}}{(2n+1)!} + \dots$?

(A) 1

(B) 0

(C) -1

(D) e

(E) There is no sum.

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Part B consists of 17 questions that will be answered on side 2 of the answer sheet. Following are the directions for Section I, Part B.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING SIDE 2 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 29–45.

YOU MAY NOT RETURN TO SIDE 1 OF THE ANSWER SHEET

In this test:

- (1) The *exact* numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Note: Question numbers with an asterisk (*) indicate a graphing calculator-active question.

29. The first three non-zero terms in the Taylor series about $x = 0$ for $f(x) = \cos x$

(A) $x + \frac{x^3}{3!} + \frac{x^5}{5!}$

(B) $x - \frac{x^3}{3!} + \frac{x^5}{5!}$

(C) $1 - \frac{x^2}{2!} + \frac{x^4}{4!}$

(D) $1 - \frac{x^2}{2!} - \frac{x^4}{4!}$

(E) $1 + \frac{x^2}{2!} + \frac{x^4}{4!}$

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30. $\int \cos^3 x \, dx =$

(A) $\frac{\cos^4 x}{4} + C$

(B) $\frac{\sin^4 x}{4} + C$

(C) $\sin x - \frac{\sin^3 x}{3} + C$

(D) $\sin x + \frac{\sin^3 x}{3} + C$

(E) $\sin^3 x + C$

31. If $f(x) = (3x)^{(3x)}$ then $f'(x) =$

(A) $(3x)^{(3x)}(3\ln(3x) + 3)$

(B) $(3x)^{(3x)}(3\ln(3x) + 3x)$

(C) $(9x)^{(3x)}(\ln(3x) + 1)$

(D) $(3x)^{(3x-1)}(3x)$

(E) $(3x)^{(3x-1)}(9x)$

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32. To what limit does the sequence $S_n = \frac{3+n}{3^n}$ converge as n approaches infinity?

(A) 1

(B) $\frac{1}{3}$

(C) 0

(D) ∞

(E) 3

33. $\int \frac{18x-17}{(2x-3)(x+1)} dx =$

(A) $8\ln(2x-3) + 7\ln(x+1) + C$

(B) $2\ln(2x-3) + 7\ln(x+1) + C$

(C) $4\ln(2x-3) + 7\ln(x+1) + C$

(D) $7\ln(2x-3) + 2\ln(x+1) + C$

(E) $\frac{7}{2}\ln(2x-3) + 4\ln(x+1) + C$

34. A particle moves along a path described by $x = \cos^3 t$ and $y = \sin^3 t$. The distance that the particle travels along the path from $t = 0$ to $t = \frac{\pi}{2}$ is

(A) 0.75

(B) 1.50

(C) 0

(D) -3.50

(E) -0.75

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35. The sale price of an item is $800 - 35x$ dollars and the total manufacturing cost is $2x^3 - 140x^2 + 2,600x + 10,000$ dollars, where x is the number of items. What number of items should be manufactured in order to optimize the manufacturer's total profit?

(A) 35 (B) 25 (C) 10 (D) 15 (E) 20

36. The area enclosed by the polar equation $r = 4 + \cos \theta$, for $0 \leq \theta \leq 2\pi$, is

(A) 0 (B) $\frac{9\pi}{2}$ (C) 18π (D) $\frac{33\pi}{2}$ (E) $\frac{33\pi}{4}$

37. Use the trapezoid rule with $n = 4$ to approximate the area between the curve $y = x^3 - x^2$ and the x -axis from $x = 3$ to $x = 4$.

(A) 35.266 (B) 27.766 (C) 63.031 (D) 31.516 (E) 25.125

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38. If $f(x) = \sum_{k=1}^{\infty} (\cos^2 x)^k$, then $f\left(\frac{\pi}{4}\right)$ is

(A) -2

(B) -1

(C) 0

(D) 1

(E) 2

39. The volume of the solid that results when the area between the graph of $y = x^2 + 2$ and the graph of $y = 10 - x^2$ from $x = 0$ to $x = 2$ is rotated around the x -axis is

(A) $2\pi \int_0^2 y(\sqrt{y-2}) dy + 2\pi \int_0^2 y(\sqrt{10-y}) dy$

(B) $2\pi \int_2^6 y(\sqrt{y-2}) dy + 2\pi \int_6^{10} y(\sqrt{10-y}) dy$

(C) $2\pi \int_2^6 y(\sqrt{y-2}) dy - 2\pi \int_6^{10} y(\sqrt{10-y}) dy$

(D) $2\pi \int_0^2 y(\sqrt{y-2}) dy - 2\pi \int_0^2 y(\sqrt{10-y}) dy$

(E) $2\pi \int_0^2 y(\sqrt{10-y}) dy - 2\pi \int_0^2 y(\sqrt{y-2}) dy$

40. $\int_0^4 \frac{dx}{\sqrt{9+x^2}} =$

(A) $\ln 3$

(B) $\ln 4$

(C) $-\ln 2$

(D) $-\ln 4$

(E) Undefined

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41. The rate that an object cools is directly proportional to the difference between its temperature (in Kelvins) at that time and the surrounding temperature (in Kelvins). If an object is initially at 35K, and the surrounding temperature remains constant at 10K, it takes 5 minutes for the object to cool to 25K. How long will it take for the object to cool to 20K?

(A) 6.66 min. (B) 7.50 min. (C) 7.52 min. (D) 8.97 min. (E) 10.00 min.

42. $\int e^x \cos x \, dx =$

(A) $\frac{e^x}{2}(\sin x + \cos x) + C$

(B) $\frac{e^x}{2}(\sin x - \cos x) + C$

(C) $\frac{e^x}{2}(\cos x - \sin x) + C$

(D) $2e^x(\sin x + \cos x) + C$

(E) $e^x(\sin x + \cos x) + C$

43. Two particles leave the origin at the same time and move along the y -axis with their respective positions determined by the functions $y_1 = \cos 2t$ and $y_2 = 4 \sin t$ for $0 < t < 6$. For how many values of t do the particles have the same acceleration?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

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44. The minimum value of the function $y = x^3 - 7x + 11$; $x \geq 0$ is approximately

- (A) 18.128 (B) 9.283 (C) 6.698 (D) 5.513 (E) 3.872
-

45. Use Euler's method with $h = 0.2$ to estimate $y(1)$, if $y' = y$ and $y(0) = 1$.

- (A) 1.200 (B) 2.0746 (C) 2.488 (D) 4.838 (E) 9.677
-

STOP

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS SECTION

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO

MAKE SURE YOU HAVE PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET AND HAVE WRITTEN AND GRIDDED YOUR NUMBER CORRECTLY IN SECTION C OF THE ANSWER SHEET

CALCULUS BC

SECTION II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

SHOW ALL YOUR WORK. Indicate clearly the methods you use because you will be graded on the correctness of your methods as well as on the accuracy of your final answers. If you choose to use decimal approximations, your answer should be correct to three decimal places.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. Two particles travel in the xy -plane. For time $t \geq 0$, the position of particle A is given by $x = t + 1$ and $y = (t + 1)^2 - 2t - 2$, and the position of particle B is given by $x = 4t - 2$ and $y = -2t + 2$.
 - (a) Find the velocity vector for each particle at time $t = 2$.
 - (b) Set up an integral expression for the distance traveled by particle A from time $t = 1$ to $t = 3$. Do not evaluate the integral.
 - (c) At what time do the two particles collide? Justify your answer.
 - (d) Sketch the path of both particles from time $t = 0$ to $t = 4$. Indicate the direction of each particle along its path.
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2. Let f be the function given by $f(x) = 2x^4 - 4x^2 + 1$.
 - (a) Find an equation of the line tangent to the graph at $(2, 17)$. Verify your answer.
 - (b) Find the x and y -coordinates of the relative maxima and relative minima.
 - (c) Find the x -coordinates of the points of inflection. Verify your answer.
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3. Water is draining at the rate of $48\pi\text{ft}^3/\text{sec}$ from a conical tank whose diameter at its base is 40 feet and whose height is 60 feet.
- (a) Find an expression for the volume of water (in ft^3) in the tank in terms of its radius.
 - (b) At what rate (in ft/sec) is the radius of the water in the tank shrinking when the radius is 16 feet?
 - (c) How fast (in ft/sec) is the height of the water in the tank dropping at the instant that the radius is 16 feet?
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4. Let f be the function given by $f(x) = e^{-4x^2}$
- (a) Find the first four nonzero terms and the general term of the power series for $f(x)$ about $x = 0$.
 - (b) Find the interval of convergence of the power series for $f(x)$ about $x = 0$. Show the analysis that leads to your conclusion.
 - (c) Use term-by-term differentiation to show that $f'(x) = -8xe^{-4x^2}$
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5. Let R be the region enclosed by the graphs of $y = 2 \ln x$ and $y = \frac{x}{2}$, and the lines $x = 2$ and $x = 8$.
- (a) Find the area of R .
 - (b) Set up, but do not integrate, an integral expression, in terms of a single variable, for the volume of the solid generated when R is revolved about the x -axis.
 - (c) Set up, but do not integrate, an integral expression, in terms of a single variable, for the volume of the solid generated when R is revolved about the line $x = -1$.
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6. Let f and g be functions that are differentiable throughout their domains and that have the following properties:

(i) $f(x + y) = f(x)g(y) + g(x)f(y)$

(ii) $\lim_{a \rightarrow 0} f(a) = 0$

(iii) $\lim_{h \rightarrow 0} \frac{g(h) - 1}{h} = 0$

(iv) $f'(0) = 1$

(a) Use L'Hopital's rule to show that $\lim_{a \rightarrow 0} \frac{f(a)}{a} = 1$.

(b) Use the definition of the derivative to show that $f'(x) = g(x)$.

(c) Find $\int \frac{g(x)}{f(x)} dx$

END OF EXAMINATION